



# Selection of Value-at-Risk Model and Management of Risk Using Information Transmission

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## Abstract

Major crises in many of the international financial institutions in the 1990's due to adverse market moves and poor internal management raised many questions about the practice of risk management all over the world. The need for a fool proof system of measuring risk was felt by practitioners of financial profession. The concept of Value-at-Risk (VaR) was the academician's answer for this challenge. This study made an attempt to analyze the performance of the techniques of VaR through subjecting their prediction to elaborate back testing. The analysis was carried out in the back ground of Indian capital market using the Nifty and the Nifty Junior daily return as the data. The study used the Garch and Tgarch models and found that the Tgarch model performed better than the Garch model in prediction of VaR. Further, a Vector Auto Regression framework applied to the two indices showed that, Nifty, the broader of the two markets, led in the case of generation of risk in the system.



## 1. Introduction

Certainty is important for smooth transaction of business. But every business normally is risky i.e. uncertain. It is especially so in financial markets. There are mainly 4 risks in financial markets, viz. credit risk, operational risk, liquidity risk and market risk. Credit risk arises when the counter party in an agreement fails to honor its commitment and is the loss associated with that breach of contract. Operational risk involves the errors in making payments or settling transactions, and includes the risk of fraud and regulatory risks. Liquidity risk is caused by an unexpected large and stressful negative cash flow over a very short period. Finally, market risk is the loss to an investment portfolio due to the adverse changes in price of the financial assets and liabilities or it is a risk caused solely by market conditions.

With the increasing activity in the financial market, specifically the stock markets, volatility and therefore the market risk of exposure has also grown to be sizable. One method of assessing the market risk of a portfolio is through value-at-risk (VaR). It is the maximum expected loss that a portfolio can incur over a certain period of time with a particular level of confidence.

A large quantity of research has already been done on the risk front using parametric and non-parametric methods. The latter generally focuses on the extreme behavior of the market and hence is more suitable to a highly volatile period. Among parametric methods those with a normally distributed error terms are the most popular ones. But generally the tails of financial return distributions are fatter. Due to this reason student's-t and GE distributions are also favored. The paper assumed a normal distribution and used GARCH and TGARCH models for the estimation of variance of returns.

The next section deals with some of the important studies in the past and in section 2, objectives and methodology are dealt with. It is followed by the empirical analysis in section 3 and concluding remarks are given in section 4.

### 1.1 Trend of Studies in the Past

In one of the studies of VaR on the Indian stock market, Varma (1999) assumes a GED (Generalized Error Distribution) and uses GARCH (GED), EWMA (GED) and EWMA (RM) models to estimate VaR. The paper computes nominal coverage, i.e., the ratio of number exceedences to the total number of observations and the comparing it with the true coverage,  $p$ . where,



P, is the level of confidence

The study preferred the use of GARCH (GED) over the other two on the basis of the results.

In another study of Nifty and S&P 500, Sarma et al (2001), used four models (GARCH, EWMA, Risk Metrics-RM, and Historical Simulation-HS) and their different variations, on the basis of the number of data points used, under the assumption of normally distributed errors to find out that GARCH and RM fares well with the latter having a slight edge. They used back testing methods for performance assessment of various models by testing for conditional and unconditional coverage and independence that were perfected by Christoffersen (1998) as well as loss functions developed by Lopez (1998).

In a study of the indices of the five of the developed countries, Angelidis et al (2004), used three variations of GARCH model (naïve, EGARCH, TGARCH) and various order of AR processes on normal GED and t-distributions. They also tested for conditional and unconditional coverage using Christoffersen's method but could not point out any model as the "best" model. They found student's-t distribution to be capturing the risk better than other distributions.

In their paper titled, 'evaluating predictive performance of Value-at-Risk models in emerging markets: a reality check', Bao et al checked the performance of VaR models in terms of empirical coverage taking parametric and non-parametric models. They used normal, historical simulation, Monte Carlo simulation non-parametrically estimated distribution and the extreme value distribution together with RM as bench mark. They analyzed the model performance before, during and after the Asian financial crisis. In the pre-crisis period RM was found to be quite good model with normal not being far behind. HS, NP and MC were also seen to be satisfactory. During the crisis all models understated the VaR numbers but the EVT based one did the best job. The post crisis period results were found to be similar to the pre-crisis period result. From the study it's clear that the conventional models do a good job during normal periods.

Nath and Samantha in their paper, 'Value-at Risk: concept and implementation for Indian banking system', studied the VaR for the Indian banking system. They used one day return on the Government of India securities as the variable. The models used were normal, historical simulation, Risk Metrics and Hill's estimator and found that VaR models under variance-covariance/normal approach, particularly Risk Metrics underestimated VaR numbers. The GARCH (normal) performed



slightly better than the Risk Metrics. HS provided quite reasonable estimates. But Hill's estimator overestimated the VaR numbers as the number of failures was less than theoretical expectation. All in all, there was no clear winner emerging from the study.

### 1.3 Motivation for the Study

From a brief survey of literature, not many studies on the model selection for VaR based on the Indian market could be traced. Researchers have mostly concentrated on the simple GARCH model. In whatever studies that have been done, GARCH and RM are seen to be performing better under normal market conditions. In this study I am attempting a VaR model selection that tests two GARCH variations (simple GARCH, TGARCH). The window period considered is 1000 days. These models will be tested against the Nifty and Nifty junior. The result will be subjected to back testing developed by Christoffersen (1998). One of the most attractive features of the study is the inclusion of the Nifty junior, the risk profile of which has not been given much attention. In the two other studies on Indian market referred to above, one has focused only on the GED and one on normal distribution and only one study has done the back testing developed by Christoffersen.

## 2. Objectives

The objectives of the study are as follows.

1. Selection of the best performing VaR model in the face of solid back testing
2. Checking whether there exist any interlinkages in the VaR figures for the different indices. This result can be used to ascertain the origin of risk in the Indian market.

### 2.1 Methodology

(a) The GARCH family of models pioneered by Engle (1982) and later developed by Bollerslev (1986) estimate conditional volatility. The simple AR (1) GARCH (1, 1) model is as follows;

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t \quad \dots\dots (1)$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t = \alpha + \beta \varepsilon_{t-1} + \gamma \sigma_{t-1} + \varepsilon_t \quad \dots\dots (2)$$



Here, the first equation is the mean equation and the second equation estimate the standard deviation (SD) as a function of previous period's error term and previous period's SD. After obtaining the conditional SD, VaR is found out under the assumption of normal distribution. The study takes two levels of confidence, viz. 1 and 5. The VaR figure is obtained by taking the area under the normal distribution and multiplying it with the SD.

(b) The naïve GARCH model discussed above has the empirical limitation that it fails to capture asymmetry reflected in stock movement as a function of the nature of information, i.e. the tendency of volatility to increase more as a result of a negative news ( $\varepsilon_t < 0$ ) than to a positive news ( $\varepsilon_t > 0$ ). This is an important observed phenomenon, and hence the paper uses the asymmetric GARCH, which is called threshold GARCH or TGARCH - a widely used model to capture asymmetry in the return distribution.

$$\sigma_t = a + \sum a_i \varepsilon_{t-1} + \gamma_1 \varepsilon_{t-1}^2 d_{t-1} + \sum \sigma_{t-j}^2 + \varepsilon_t, \dots (3)$$

where,

$d_t$  is the dummy which is equal to one if  $\varepsilon_t < 0$ , and zero otherwise.

After obtaining the conditional SD, VaR is found out for normal distribution using the above discussed method.

For both the above models a 5 and 1 per cent levels of confidence are used. The conditional SD is obtained using as many data points as necessary and in this process we move forward one day at a time.

Eg: For a window period of 1000 observations, 1001<sup>st</sup> day's VaR forecast is made using the first 1000 observations. For the 1002<sup>nd</sup> day's VaR forecast, first day is dropped and return from 2<sup>nd</sup> to 1001<sup>st</sup> are used and this process goes on until we have sufficient number observations (the study is using 1500 such one step ahead forecasts) for back testing purposes.

The equations for conditional volatility predictions of the two models are given below.

#### GARCH

$$\sigma_{t+t} = a0^{(t)} + \sum a_i^{(t)} \varepsilon_{t-i+1} + \sum b_j \sigma_{t-j+1}^2 + \varepsilon_t \dots (4)$$



## TGARCH

$$\sigma_{t+t} = \mathbf{a0}^{(t)} + \sum \varepsilon_{t+i+1}] + \gamma^{(t)} \varepsilon_{t}^2 \mathbf{d}_t + \sum \sigma_{t-j+1} + \varepsilon_t \quad \dots(5)$$

### 2.1.1 Back Testing

(A) Back testing of estimated VaR figures is done on the basis of the philosophy that the failure rate in the correctly specified VaR model should not exceed the pre-specified failure rate. For this purpose Peter F Christoffersen (1998), has developed three tests, .viz.

1. Test of correct unconditional coverage
2. Test of independence
3. Test of correct conditional coverage

These tests as well as the loss function which is to follow were extensively used by Sarma et al (2001) and Angelidis et al (2004) in their papers.

For back testing christoffersen defines failure rate It as;

$$I_t = \begin{cases} 1 & \text{if } r_t < v_t \\ 0 & \text{otherwise, where,} \end{cases}$$

$v_t$  is the forecast value of VaR. The tests are carried out in the likelihood ratio (LR) framework the statistic of which is;

#### (1) Correct Unconditional Coverage Test

Here the null hypothesis is that the failure probability is p.

$$LR_{uc} = -2 \log [p^{n_1}(1-p)^{n_0} \pi^{n_1}(1-\pi)^{n_0}] \sim \chi^2(1) \quad \dots(6)$$

P: confidence level

$n_1$ : number of failures, i.e. VaR exceedences

$n_0$ : number of successes

$\pi$ :  $n_1 n_0 + n_1$ , the MLE of p.



## (2) Test of Independence

In the test for independence, the hypothesis of an independently distributed failure process is tested against the alternative of a Markov first order failure process.

$$LR_{ind} = -2 \log (1-\pi_2)^{(n_{00}+n_{10})} \pi_2^{(n_{01}+n_{11})} (1-\pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}} \sim \chi^2(1) \quad \dots(7)$$

Where,

$n_{ij}$  = number of  $i_s$  followed by number of  $j_s$

$\pi_{ij} = \Pr \{I_t = i / I_{t-1} = j\} \quad (i, j=0)$

$\pi_{01} = n_{01} / n_{00} + n_{01}$

$\pi_{11} = n_{11} / n_{10} + n_{11}$

$\pi_2 = n_{01} + n_{11} / n_{00} + n_{01} + n_{10} + n_{11}$

## (3) Test of Correct Conditional Coverage

In the correct conditional coverage, the null hypothesis of an independent failure

Process with failure probability  $p$  is tested against the alternative hypothesis of first

Order Markov process with a failure probability that is different from  $p$ .

$$LR_{cc} = -2 \log (1-p)^{n_0} p^{n_1} (1-\pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}} \sim \chi^2(1) \quad \dots(8)$$

For the second objective of finding the Interlinkage of value at risk among the selected indices, the paper uses Vector Auto Regression method. This will help us to conclude how much the VaR of one market is influenced by that of the other. Equation 9 and 10 represents the models.



$$I_{1t} = \alpha + \alpha_1 I_{-1} + \alpha_2 I_{j-1} + \varepsilon_a \dots\dots\dots (9)$$

$$I_{j1t} = \beta + \beta_1 I_{-1} + \beta_2 I_{j-1} + \varepsilon_a \dots\dots\dots (10)$$

$I, I_j$ , refers to the VaR figures of Nifty and Nifty Junior.

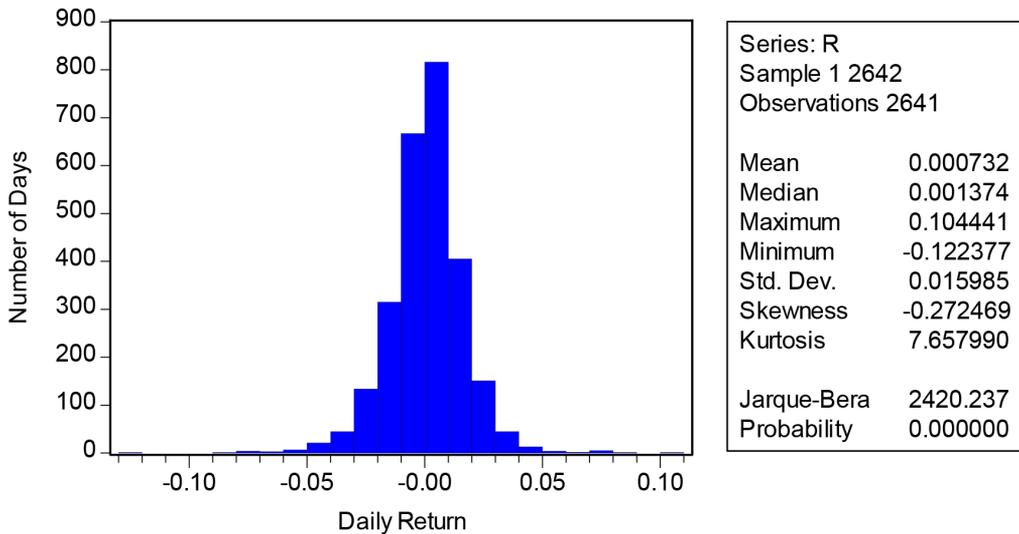
Using the VaR framework, we can see how many the innovations in VaR in one market affects the other one as also the response of one market to an impulse or shock in another market.

### 2.2 Data

The data for this work has been solely extracted from the NSE official website and ranges from 1<sup>st</sup> January 1997 to 20<sup>th</sup> July 2007. First part of this range is used for prediction and the next part is used fro comparison of results on a rolling basis.

### 3 Empirical Analysis

Before going in for the empirical analysis it may be useful to have preliminary analysis of the data.

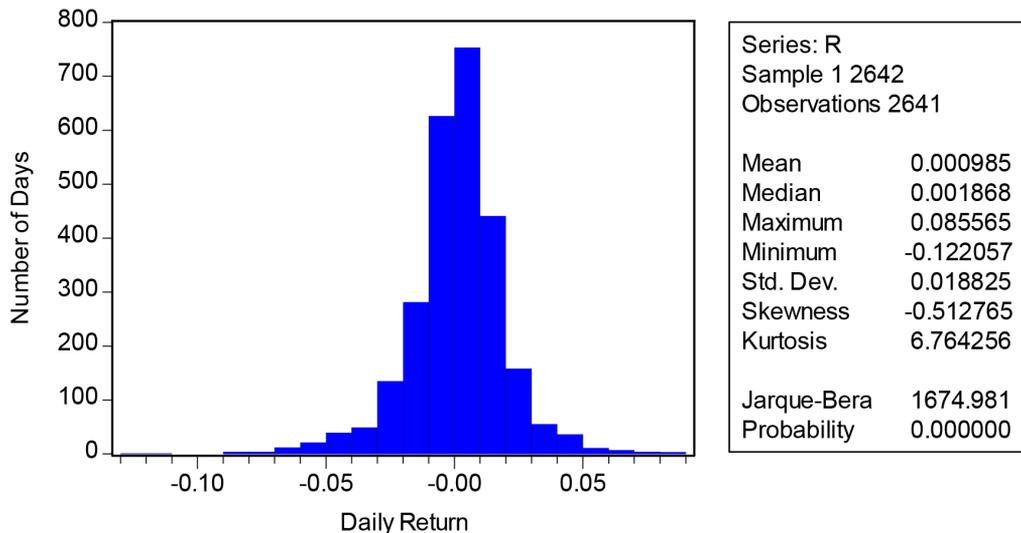


**Figure-1** Distribution of Nifty Returns



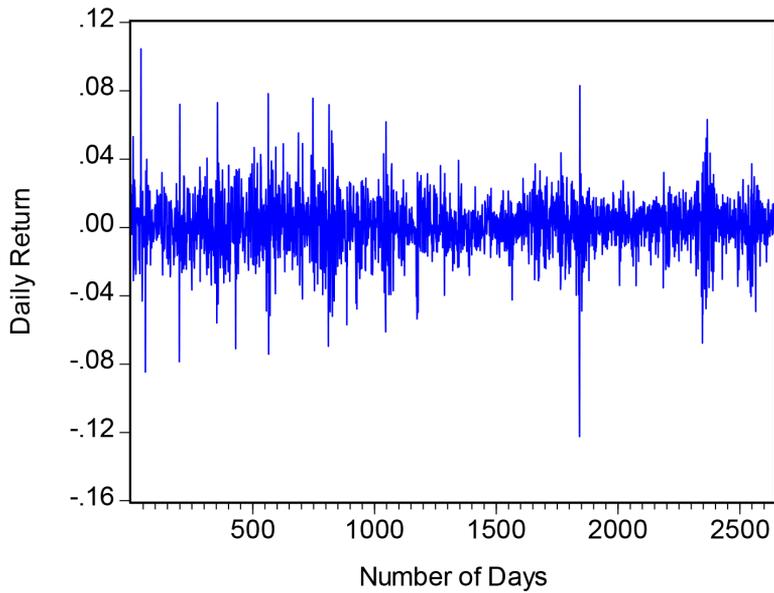
Nifty returns and the Jarque-Bera statistics show that the distribution of the returns is not normal though a cursory glance at the figure may show that it is normal. From the kurtosis and skewness figures it is clear that the distribution is leptokurtic and negatively skewed. The return series is shown in a different graph to get a clear picture of volatility of the returns.

A similar exercise is done for the Nifty Junior return series.

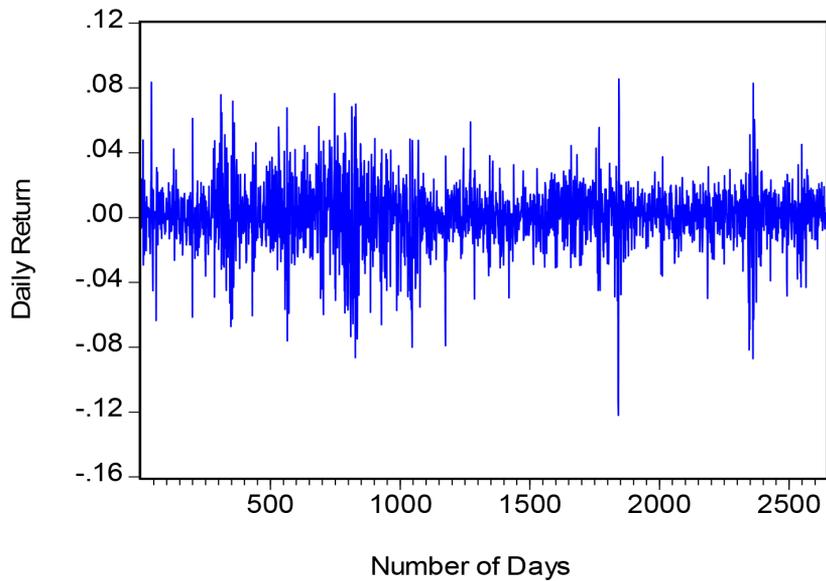


**Figure 2** Distribution of Nifty Junior Returns

The returns of Nifty followed by that of the Nifty junior is given which clearly shows the volatility clustering in the data, which justifies the selection of volatility models for the prediction of VaR. Further, the GARCH models are also capable of accounting for the fat tails to some extent. In addition a formal test-Engle's Arch test- to find out existence of arch effect in the data was performed which showed a positive outcome. The results are given in appendix 1 and 2. Appendix 3 shows the result of Augmented Dickey-Fuller test which confirms that the data is stationary.



**Figure 3.** Volatility Clustering in Nifty returns



**Figure 4.** Volatility Clustering in Nifty junior returns

**Table 1.** Back Testing for GARCH models

Confidence level→	Nifty		Nifty Junior	
	95%	99%	95%	99%
No: exceedences	51	0	49	0
$LR_{uc}$	8.03*	NA	10.76*	NA
$LR_{ind}$	2.35	NA	4.80**	NA
$LR_{cc}$	11.42*	NA	9.75*	NA

\* Significant at 1% level

\*\* Significant at 5% level

The results for GARCH models show that VaR at one percent level of significance though gives no exceedences in both the markets is not a good one because it over estimates the risk which will raise the opportunity cost of the institutions using it. In all the cases except the Nifty 95% level the null hypothesis is rejected. It means that the exceedence level is significantly different from the theoretical value thereby recommending against the use of these models.

**Table 2.** Back Testing for TGARCH models

Confidence level→	Nifty		Nifty Junior	
	95%	99%	95%	99%
No: exceedences	79	6	50	1
$LR_{uc}$	.22	110.97*	9.89*	22.61*
$LR_{ind}$	1.97	18.34*	2.57	16.72*
$LR_{cc}$	8.09**	111.05*	12.42*	5.89*

\* Significant at 1% level

\*\* Significant at 5% level



In the case of TGARCH, the Nifty result covers the Value-at-Risk well in the 95% level with the null hypothesis being accepted for unconditional coverage and independence but lacks conditional coverage. But for Nifty Junior, all tests except the independence test fails to affirm the effectiveness of TGARCH modeling.

From a comparison of the result, it's clear that the TGARCH model estimated at the 95% level of confidence is the better one for VaR calculation as compared to the simple GARCH and the 99% confidence level of both variations.

### 3.1 Interlinkage of Risk

For analyzing the Interlinkage of risk between the two markets the study has used the Vector Auto Regression (VAR) frame work with one lag of each variable (Nifty and Nifty junior VaR) forming the endogenous variable part of the vector. Vector Auto Regression is a technique capable of showing causality and leadership among variables. If the coefficient of Nifty SD deviation was found to be substantial and statistically significant, it is a proof that risk, as measured by the standard deviation here, is caused in Nifty and flows to Nifty Junior. This will add to the intuitive belief that Nifty is the more important of the two indices and vice versa. The lag structure was arrived at after testing the Akaike and Schwarz information criteria. The series were subjected to augmented Dicky-Fuller test and was found to stationery. So there is no need to go for co-integration test. The result for stationarity test is given appendices 3.

**Table 3.** Interlinkage of risk using VaR

	Nifty VaR	Nifty Junior VaR
Nifty VaR <sub>-1</sub>	.744	2.029
t- value	34.51	2.61
Nifty Junior VaR <sub>-1</sub>	.004	.876
t- value	8.01	45.75
R-squared	0.80	0.84
Adjusted R-squared	0.80	0.84
F statistic	3032.03	4049.99



The result shows that risk in both the markets are mutually affected by the other market. The R-squared, t-value and the F statistic lends credibility to the result. But the influence of Nifty Junior on Nifty was found to be significant as the coefficient is only 0.2.029 and t-statistic is 2.61 whereas the Nifty Junior coefficient is only 0.876. Since this result does not provide any conclusive result as to which market is having a say on the other it would be more useful to look at the impulse responses and the variance decompositions, which tells the impact on a variable of a unit shock on that and the other variable and the percentage of movement in a variable as its own function and as a function of the second variable respectively. If one variable is found to have a greater percentage influence on the other, then that one can be identified as the generator of risk in the system and thereby the risk profile of the follower market can be gauged to some extent from the information available from the leading market. Below is given the 5 period impulse responses with the ordering NR (Nifty), NRJ (nifty Junior).

**Table 4. Impulse Responses**

Responses of NR			Responses of NRJ		
Period	NR	NRJ	Period	NR	NRJ
1	0.00399	0	1	6.204e-05	9.204e-05
2	0.00362	0.00018	2	6.324e-05	6.853e-05
3	0.00330	0.00030	3	6.256e-05	5.182e-05
4	0.00302	0.00037	4	6.068e-05	3.987e-05
5	0.00277	0.00040	5	5.808e-05	3.127e-05

Here, NR signifies Nifty risk as measured by Nifty VaR and NRJ signifies Nifty Junior VaR. Since, the ordering of variables is important in case of impulse responses and variance decomposition, both type of ordering were tried and the one that appeared theoretically more appealing was selected. The impulse responses were found significant also. From the results it appears that the shocks to the innovations are not very important in the case of either market with the best one being one unit shock to the Nifty return leading to a 0.00399 unit increase in the VaR figure for Nifty the next day which only comes to 0.39 percent.

**Table 5.** Variance Decomposition

Variance Decomposition of NR				Variance Decomposition of NRJ			
Period	SE	VF	VFJ	Period	SE	VF	VFJ
1	0.0039	100.00	0.0000	1	0.0001	31.241	68.758
2	0.0054	99.885	0.1194	2	0.0001	37.344	62.655
3	0.0063	99.684	0.3144	3	0.0001	42.591	57.408
4	0.0070	99.467	0.5325	4	0.0001	46.960	53.039
5	0.0075	99.253	0.7460	5	0.0001	50.532	49.467

As per the results, while most of the movement in the Nifty risk is explained by the shocks on itself, 31% of the movement in the risk of Nifty Junior is explained by the shock on Nifty risk. This figure goes on climbing and in to the 5<sup>th</sup> period the percentage of movement in the Nifty Junior risk is explained by the shocks on Nifty risk by a whopping 50%. From this it is clear that the risk is actually flowing from the Nifty to the Nifty Junior and the risk of Nifty Junior can to some extent be anticipated by the shocks on Nifty risk.

### Policy Implications

**In the light of the out performance of TGARCH models over GARCH it is obvious that any model, whether used by policy makers or risk industry analysts, should account for the asymmetry present in financial data. Hence, TGARCH model scores over many other models in this respect as per the finding of the present study. Hence, it is recommended that financial planners and policy makers should apply TGARCH models more often to get more accurate results.**

### 4. Concluding Remarks.

The models covered in the present context are the GARCH and TGARCH. Window size is one dimensional .i.e. 1000. But out of this study, it is obvious that the GARCH models do a good job in predicting the Value-at-Risk of the Nifty and Nifty Junior markets though they can not be fully relied on to make decisions as in most cases they over estimate the risk. An expansion of the study with different distributional assumptions from the normal may be conducted and the results compared with this study to see how different the findings are when the distributional assumptions change as it is well known that the distribution of financial return series are fatter than normal. Of the two models considered, the



TGARCH is more appropriate from an all round perspective. This may be due to the asymmetry present in the return data. The confidence level found to the better is the 95% level. The 99% level may be good from the regulatory point of view but it does not give the institutions full free play –with the presence of some opportunity cost. The study of the interlinkage of risk of the two markets using the Vector Auto Regression frame work revealed that the risk is flowing from the Nifty to the Nifty Junior emphasizing the dominant role played by the former. Variance decomposition also supports the same view.

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### Appendix 1: Arch Test (Engle) -Nifty

Lags	'p' Value	Chi Square Value	Critical Value
1	0	3.6326e+002	3.8415e+000
2	0	3.6976e+002	5.9915e+000
3	0	3.7371e+002	7.8147e+000
4	0	3.7869e+002	9.4877e+000
5	0	3.7856e+002	1.1070e+001
6	0	3.7831e+002	1.2592e+001
7	0	3.7811e+002	1.4067e+001
8	0	3.8008e+002	1.5507e+001
9	0	3.8955e+002	1.6919e+001
10	0	3.9039e+002	1.8307e+001

### Appendix 2: Arch Test (Engle)-Nifty Junior

Lags	'p' Value	Chi Square Value	Critical Value
1	0	4.3895e+002	3.8415e+000
2	0	4.4099e+002	5.9915e+000
3	0	4.5005e+002	7.8147e+000
4	0	4.5077e+002	9.4877e+000
5	0	4.5283e+002	1.1070e+001
6	0	4.5252e+002	1.2592e+001
7	0	4.5279e+002	1.4067e+001
8	0	4.5498e+002	1.5507e+001
9	0	4.6131e+002	1.6919e+001
10	0	4.6321e+002	1.8307e+001

### Appendix 3: Augmented Dicky -Fuller Test

Name	t-Statistic	1% critical value	5% critical value	10 % Critical Value
Nifty	6.42	3.43	2.86	2.56
Nifty Junior	6.23	3.43	2.86	2.56