ANALYTIC SOLUTION OF NONLINEAR BOUNDARY VALUE PROBLEM GOVERNING THE ELECTRICAL BEHAVIOUR OF CELL MEMBRANE

S. Baiju*

In this paper, AdomianDecomposition Method(ADM) and Taylor Expansion Approach(TEA) are applied to solve nonlinearintegro-differential equation arising in cell membrane theory. After solving theproblem acomparison is made between them.

***Keywords*:** Integro-differential equation,Adomian polynomial, Taylor polynomial, Analytical Treatments, Numerical treatments

Introduction

Mathematical formulation of scientific and engineering problems often turn out to be nonlinear initial and/or boundary value problems in nature. The interplay between applied science and mathematics leads to the development of initial and/or boundary value problems for nonlinear partial differential or integral or integro-differential equations modelling real physical systems. The theory and application of integral and integro-differential equations is important in applied mathematics. These equations are used as mathematical models for many and varied physical situations and also occur as reformulations of other mathematical problems. These problems contain some physical parameters. While solving, these parameters are assumed to be small or large. Sometimes the nonlinearity is assumed to be weak. Nonlinear boundary value problems can be solved either numerically or analytically.

Assistant Professor, Department of Mathematics, Govt. College, Kariavattom, Thiruvananthapuram, Kerala.
E-mail: baijupoint@gmail.com

Analytic solutions are important since they give physical insight to the problems under investigation,which is lacking in the numerical results. Analytic solution to nonlinear problems can be obtained in many ways such as Taylor Series Method [4], Picard's Method [10], AdomianMethod [1, 12], Taylor Expansion Approach [8], Laplace Decomposition Method [13], Reversion Method [13], Sarafyan's Method [2] and Laplace Adomian Decomposition Algorithm [7].

Taylor Series Method is difficult to apply since one need to find an analytic expression for arbitrarily high derivatives. In Picard's successive substitution method, the complexity increases very rapidly. Adomian method is an optimal method-optimal in the sense that it requires lower number of derivatives, evaluation in the nonlinear term to obtain Adomian polynomials and successive terms of the decomposition are calculated very easily and summed up to obtain an approximate analytic solution. Sarafyan's method for ordinary differential equations and systems obtain continuously differential polynomials yielding a fourth order Runge-Kutta approximation at all interior points of an interval and a fifth order approximation at the end points.

In this paper we discuss the Taylor Expansion Approach (TEA) for solving initial/boundary value problems of nonlinear integro-differential equations. This method transforms nonlinear integro-differential equation to matrix equation which corresponds to a system of nonlinear equations with unknown coefficients. These unknown coefficients can be found by the initial conditions successively to the higher approximations.

The numerical solution of nonlinear integro-differential equation has been highly studied by many authors. In recent years, numerous works have been focusing on the development of more advanced and efficient methods for integro-differential equations such as Wavelet-Galerkin Method [3], Lagrange's Interpolation Method [12] and Tau Method [9] and semi-analytical techniques such as Laplace Adomian Decomposition Algorithm [7] Adomian Decomposition Method [12]. Moreover, previous studies require more effort to achieve the results, they are not accurate and usually developed for special type of integro-differential equations.

2. Adomian Decomposition Method

Adomian method consists of

(i) splitting the given operator equation into linear and nonlinear parts

(ii) operating by the inverse of the linear operator on both sides(In most of the cases only the highest order derivative of the linear part is inverted and the rest is considered with the nonlinear part since the inversion is linear)

(i) decomposing the unknown function into a sum whose components are to be determined

(ii) identifying the terms arising out of source terms and initial and/or boundary conditions as the initial term of the sum and

(iii) obtaining the successive terms of the sum in terms of the initial term using Adomian polynomials, which are nothing but the coefficients of the powers of a parameter in the Maclaurin's series expansion of the given nonlinear function.

In Adomian Method all the conditions given in the problems were used to obtain the initial term u_0 ,irrespective of whether the given problem was initial/boundary value problems. All other components of the sum were calculated using the Adomian polynomials which dependon. Thus finding u_0 correctly and completely is an important step while applying Adomian Method. For initial value problems u_0 can be directly found out using the given initial conditions. But for boundary value problems, it was rather difficult to find completely and correctly from the conditions given. To avoid these difficulties Venkatarangan and Sivakumar introduced a modification of the method, [15] Shooting Type ADM. Further details can be found in [7], [15], [17].

3. **Taylor Expansion Approach**

General form of the problems under consideration are nonlinear integro-differential equations of Fredholm and Volterra types in the forms

$$y'(x) = f(x) + \int_a^b K(x,t) g(t, y(t))\, dt \quad ; \; a \le x \le b$$

and

$$y'(x) = f(x) + \int_0^b K(x,t)\, g(t, y(t))\, dt \quad ; \; 0 \le x \le b$$

with initial condition $y(a) = y_0$ where a and b are constants, $f(x)$, $K(x,t)$, $g(t, y(t))$ are known functions and $y(x)$ is the solution to be determined.In this equations, without loss of generality, we assume that. It will become clear that, following analysis can be readily extended to every a$\epsilon\mathcal{R}$. Hence the general form is converted into the following way

$$y'(x) = f(x) + \int_0^b K(x,t)\, g(t, y(t))\, dt \quad ; \quad 0 \le x \le b$$

and

$$y'(x) = f(x) + \int_0^x K(x,t) g(t, y(t))\, dt$$ with the initial condition $y(0) = y_0$

Consider the Volterra integro-differential equations.

Assume that the solution of the Volterra integro-differential equation is expressed in the form

$$y(x) = \sum_{n=0}^{N+1} \frac{1}{n!} y^{(n)}(0) x^n \quad \text{............ (1)}$$

which is the Taylor polynomial [16] of degree $N+1$ at $x=0$ where $y(0)=y_0$ and the coefficients $y^{(n)}(0)$; $n = 1, 2, 3, \ldots\ldots\ldots\ldots, N+1$ are to be determined.

Let $g(t, y(t)) = G(y(t)) = \sum_{l=0}^{N} W(l) Y_l(t)$ where $W(l) = \frac{1}{l!} \frac{d^l}{dx^l} (G(y(t))) \Big|_{y(t)=0}$ be the

the nonlinear term in Volterra integro-differential equation.

For $l=0$ $Y_0(t)=1$, and for $l>0$, we obtain the Taylor expansion of $Y_l(t)$ at $t=0$ in the form

$$Y_l(t) = \sum_{m=0}^{N} \frac{Y_l^{(m)}(0)}{m!} t^m$$

Substituting in the nonlinear Volterra integro-differential equation

$$y'(x) = f(x) + \sum_{l=0}^{N} W(l) \int_0^x K(x,t) Y_l(t)\, dt \quad \text{......................(2)}$$

Differentiate (2) n times with respect to x,

$$y^{(n+1)} = f^{(n)}(x) + \sum_{l=0}^{N} W(l) V^{(n)}(x) \quad \text{............(3)}$$

where

$$V^{(n)}(x) = \frac{d^n}{dx^n} \left[\int_0^x K(x,t) Y_l(t) dt \right]$$

and for, $n > 0$ by applying successively n times the Leibnitz's rule in the integral, $V(x)$

$$V^{(n)}(x) = \sum_{j=0}^{n-1} [h_j(x) Y_l(t)]^{(n-j-1)} + \int_0^x \frac{\partial^n}{\partial x^n}[K(x,t)] Y_l(t)\, dt \quad \text{......(4)}$$

where

$$h_j(x) = \frac{\partial^j}{\partial x^j}[K(x,t)]\Big|_{t=x}$$

From the Leibnitz's rule,we evaluate $[h_j(x) Y_l(t)]^{(n-j-1)}$ as

$$[h_j(x) Y_l(t)]^{(n-j-1)} = \sum_{m=0}^{n-j-1} \binom{n-j-1}{m} h_j^{(n-m-j-1)}(x) Y_l^{(m)}(x)$$

Hence $V^{(n)}(x)$ is expressed as

$$V^{(n)}(x) = \sum_{m=0}^{n-j} \sum_{j=0}^{n-m-1} \left\{ \binom{n-j-1}{m} h_j^{(n-m-j-1)}(x) Y_l^{(m)}(x) \right\} + \int_0^x \frac{\partial^n}{\partial x^n}[K(x,t)] Y_l(t)\, dt$$

Substituting in (3) and set $x = c$

$$y^{(n+1)}(c) = f^{(n)}(c) + \sum_{l=0}^{N} W(l) \left\{ \sum_{m=0}^{n-j} \sum_{j=0}^{n-m-1} \left\{ \binom{n-j-1}{m} h_j^{(n-m-j-1)}(c) Y_l^{(m)}(c) \right\} \right. + \sum_{l=0}^{N} W(l) \left\{ \int_0^c \frac{\partial^n}{\partial x^n}[K(x,t)]\Big|_{x=c} Y_l(t)\, dt \right\}$$

At $t = c$ for $l > 0$, substituting $Y_l(t)$ in the above equation and for convenience represented as

$$y^{(n+1)}(c) = f^{(n)}(c) + \sum_{l=0}^{N} \left\{ \sum_{m=0}^{n-1} \left(H_{nm,l} + T_{nm,l}\right) Y_l^{(m)}(\mathrm{c}) + \sum_{m=n}^{N} T_{nm,l} Y_l^{(m)}(\mathrm{c}) \right\}$$

For $n = 0$

$$\sum_{m=0}^{n-1} \left(H_{nm,l} + T_{nm,l}\right) Y_l^{(m)}(\mathrm{c}) = 0\ ;\ l = 0, 1, 2, 3, \ldots\ldots$$ and for , $n < m$

For $n >$ m

$$H_{nm,l} = 0\ ;\ l = 0, 1, 2, 3, \ldots\ldots\ldots\ldots, N$$

Also for $n, m, l = 0, 1, 2, 3, \ldots\ldots\ldots\ldots, N$

$$T_{nm,l} = W(l) \frac{1}{m!} \int_0^c \frac{\partial^n}{\partial x^n} [\, K(x,t)\,] \Big|_{x=c} (t-c)^m dt$$

The equation $Y_l^{(m)}(\mathrm{c})$ for $m = 0, 1, 2, 3, \ldots\ldots\ldots, N$ can be found from the permutation relation

$$Y_l^{(m)}(\mathrm{c}) = \begin{cases} \displaystyle\sum_{t_1 + t_2 + \ldots + t_l = m} \binom{m}{t_1 t_2 \ldots t_l} y^{(t_1)}(0)\, y^{(t_2)}(0) \ldots y^{(t_l)}(0) & ;\ l > 0,\ m = 0, 1, 2, \ldots, N \\ 0 & ;\ l = 0,\ m \neq 0 \\ 1 & ;\ l = m = 0 \end{cases}$$

where $\begin{pmatrix} m \\ t_1 \ t_2 \ t_3 \ ......t_l \end{pmatrix} = \dfrac{m!}{t_1!t_2!t_3!..........t_l!}$; $t_1, t_2, t_3,.........., t_l$ are positive integers or zero.

If we take, $n, m = 0, 1, 2, 3,.........., N$ (5) becomes

$$y^{(1)}(c) = f^{(0)}(c) + \sum_{l=0}^{N}\sum_{m=0}^{N} T_{0m,l}Y_l^{(m)}(c)$$

$$y^{(n+1)}(c) = f^{(n)}(c) + \sum_{l=0}^{N}\left\{\sum_{m=0}^{n-1}(H_{nm,l} + T_{nm,l})Y_l^{(m)}(c) + \sum_{m=n}^{N} T_{nm,l}Y_l^{(m)}(c)\right\}$$

whichis a nonlinear system of $N+1$equations for $N+1$ unknown $y^{(n+1)}(c)$; $n = 0, 1, 2, 3,.............., N$ which can be solved numerically by any standard method.This systemcan be written as a matrix form

$$Y - \sum_{l=0}^{N} T_l Y_l^* = F \quad ....(6)$$ where

$$Y = \begin{bmatrix} y^{(1)}(c) \\ y^{(2)}(c) \\ y^{(3)}(c) \\ ...... \\ ..... \\ .... \\ y^{(N+1)}(c) \end{bmatrix} \quad F = \begin{bmatrix} f^{(0)}(c) \\ f^{(1)}(c) \\ f^{(2)}(c) \\ ...... \\ ..... \\ .... \\ f^{(N)}(c) \end{bmatrix} \quad T_l = \begin{bmatrix} T_{00,l} & T_{01,l} & T_{02,l} & \cdots & \cdots & T_{0N,l} \\ T_{10,l}+H_{10,l} & T_{11,l} & T_{12,l} & \cdots & \cdots & T_{1N,l} \\ \cdots & \cdots & \cdots\ \cdots & \cdots & & \cdots \\ \cdots & \cdots & \cdots\ \cdots & \cdots & & \cdots \\ \cdots & \cdots & \cdots\ \cdots & \cdots & & \cdots \\ H_{N0,l} & H_{N1,l} & \cdots & \cdots & \cdots & 0 \end{bmatrix} \quad Y_l^* = \begin{bmatrix} Y_l^{(0)}(c) \\ Y_l^{(1)}(c) \\ Y_l^{(2)}(c) \\ ...... \\ ..... \\ .... \\ Y_l^{(N)}(c) \end{bmatrix} \quad Y_l^* = \begin{bmatrix} Y_l^{(0)}(c) \\ Y_l^{(1)}(c) \\ Y_l^{(2)}(c) \\ ...... \\ ..... \\ .... \\ Y_l^{(N)}(c) \end{bmatrix}$$

To make easy calculation, we set $c=0$ then $T_{nm,l}=0$ .Therefore, the system becomes

$$T_l=\begin{bmatrix} 0 & 0 & 0 & \dots & \dots & 0 \\ H_{1\,0,l} & 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ H_{N\,0,l} & H_{N\,1,l} & \dots & \dots & \dots & 0 \end{bmatrix}$$

4. **Application to the problem arising in cell membrane theory**

One dimensional cable theory is important and useful in the study of electrical properties of cell membrane, because it makes the theoretical connection between specific membrane properties and the overall electrical behaviour of an important class of membrane distributions.

Let x measure the distance down a one dimensional cable.Current is injected at the end $x=0$and the other end $x=1$is terminated in an open circuit. The electrical behaviour of the cable is governed by the following pair of differential equations

$$\frac{du}{dx}=-ri \quad ....(7)$$

$$-\frac{di}{dx}=m(u) \quad ....(8)$$ with and

Here $u(x)$ represent the transmembrane potential, $i(x)$ is the axial or longitudinal current down the core of the cable and r is the longitudinal resistance per unit length, can be linear or nonlinear function of u which crosses the $m(u)$ axis only once with a possible slope.

The above problem can be written as $\frac{d^2u}{dx^2}=rm(u)\ ;\ 0<x<1$ ....(9);

where $u(0)=1$and $i(1)=0$ ....(10,

Here we take $m(u)=u+u^3$ and $r=1$

Suppose $u'(0)=a$ and inverting the operator $\frac{d^2}{dx^2}$ in the equation (9),we obtain

$$u(x) = 1 + ax + \int_0^{x}\int_0^{x_1} m(u)\, dx_1\, dx_2 \qquad \text{..........(11)}$$

Analytic solution have been obtained by using ADM in the paper Venkatarangan, and Sivakumar, Baiju [7,14]. Numerical comparison with the existing result is also made in this paper.

In the present paper we found the analytic solution of the above problem using Taylor Expansion Approach(TEA) developed by P.Darania and A. Ebadian [8]. Numerical comparison with ADM is also made in the present paper.

The nonlinear integro-differential equation arising in cell membrane theory can be written in the form

$$\frac{du}{dx} = a + \int_0^{x} \left[u(t) + u^3(t)\right] dt \text{ with } u'(0) = a, 0 < x < 1, u(0) = 1, u'(1) = 0$$

Let $f(x) = a \quad , \quad g(t, y(t)) = G(y(t)) = u(t) + u^3(t)$

then $f^{(1)}(0) = f^{(2)}(0) = f^{(3)}(0) = \text{..........................} = f^{(N)}(0) = 0$

Also $W(1) = W(3) = 1 \quad , \quad W(0) = W(2) = W(4) = W(5) \text{ ....} W(N) = 0$

$$T_0 = T_2 = T_4 = T_5 = \text{....................} = T_N = 0$$

$$T_1 = T_3 = \begin{bmatrix} 0 & 0 & 0 & \dots & \dots & \dots & \dots & 0 \\ 1 & 0 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix}_{NxN}$$

On substitution and simplification in (6) for the case $N = 4$,we have

$$u^{(1)}(0) = a\ ,\ u^{(2)}(0) = 2\ ,\ u^{(3)}(0) = u^{(1)}(0) + 3u^{(1)}(0) \Rightarrow u^{(2)}(0) = 4\,a$$

$$u^{(4)}(0) = u^{(2)}(0) + 3u^{(2)}(0) + 6\left(u^{(1)}(0)\right)^2 \Rightarrow u^{(4)}(0) = 8 + 6a^2$$

$$u^{(5)}(0) = u^{(3)}(0) + 3u^{(3)}(0) + 18u^{(1)}(0)u^{(2)}(0) + 6\left(u^{(1)}(0)\right)^3 \\ \Rightarrow u^{(2)}(0) = 52\,a + 6a^3$$

Approximate analytic solution at this stage is

$$u(x) = 1 + \frac{1}{1!}ax + \frac{1}{2!}2x^2 + \frac{1}{3!}4ax^3 + \frac{1}{4!}(8 + 6a^2)x^4 \\ + \frac{1}{5!}(52\,a + 6a^3)x^5$$

$$u'(1) = 0 \Rightarrow \boldsymbol{a = -0.7293455514}$$

For higher approximation at $N = 28$, approximate analytic solution is

$$u(x) = 1 + \frac{1}{1!}ax + \frac{1}{2!}2x^2 + \frac{1}{3!}4\,ax^3 + \frac{1}{4!}(8 + 6\,a^2)x^4 \\ + \frac{1}{5!}(52\,a + 6\,a^3)x^5 + \frac{1}{6!}(104 + 192\,a^2)x^6 + \frac{1}{7!}(1288\,a \\ + 444\,a^3)x^7 + \frac{1}{8!}(2576 + 8280\,a^2 + 756\,a^4)x^8 \\ + \frac{1}{9!}(49504a + 37224a^3 + 756\,a^5)x^9 + \frac{1}{10!}(99008 \\ + 509568a^2 + 124632a^4)x^{10} + \frac{1}{11!}(2808736a + 3641136a^3 \\ + 307584a^5)x^{11} + \frac{1}{12!}(5617472 + 42076416a^2$$

$$+ 19703952a^4 + 548856a^6)x^{12} + \frac{1}{13!}(219999808a + 435473472a^3 + 84057696a^5 + 548856a^7)x^{13} + \frac{1}{14!}(4508267904a^2 + 3450667392a^4 + 282375072a^6 + 439999616)x^{14} + \frac{1}{15!}(22703924992a + 63766620288a^3 + 21911835744a^5 + 724553424a^7)x^{15} + \frac{1}{16!}(1326535056a^8 + 610344518784a^2 + 691984105920a^4 + 113621154624a^6 + 45407849984)x^{16} + \frac{1}{17!}(2987683369216a + 1326535056a^9 + 11279390817408a^3 + 6035248567872a^5 + 48216346156a^7) x^{17} + \frac{1}{18!}(44144335224576a^6 + 101977600134144a^2 + 1645231204512a^8 + 160472361437568a^4 + 5975366738432) x^{18} + \frac{1}{19!}(472085383565824a + 235420382796211a^3 + 182140008762700a^5 + 262548546573312a^7 + 4193699940864a^9)x^{19} + \frac{1}{20!}(806222741385a^{10} + 2062422525758515a^2 + 4300353159167923a^4 + 1385260865968896a^8 + 976519294542848 + 1782054695138803a^6)x^{20} + \frac{1}{21!}(959522901077278a + 599535022541672448a^3 + 625492403805217536a^5 + 772517476894867a^9 + 14094054280288166a^7 + 16302382588578244a^{11} + 32945548027392a^{13})x^{21} + \frac{1}{22!}(251060348802830336 + 500804358155566080a^2 + 147349868649984a^{12} + 760367894708574a^{10} + 1207292834637475200a^8 + 7442970319836937728a^6 + 1237125276061413580a^4)x^{22} + \frac{1}{23!}(229842762489114238a + 7907188131623516716a^7 + 176248847282916613632a^3 + 233100195548558567040a^5 + 131782192109568a^{13} + 611216021475707184a^9$$

$$+ 58656102109710336a^{11})x^{23} + \frac{1}{24!}(9.30359785610^{20}\, a^{8}$$
$$+ 3.41955471810^{21}\, a^{6} + 1.38065993310^{21}\, a^{2}$$
$$+ 4.49426376810^{21}\, a^{4} + 3.12909417510^{19}\, a^{10}$$
$$+ 462866938612485693 44+ 611986068365266425 6a^{12})x^{24}$$

$$u'(1) = 0 \Rightarrow \boldsymbol{a = -1.021515903}$$

Using the second condition $u'(1) = 0$, we get approximation to $u'(0)$,which in turn should be substituted in the analytic solution obtained in various values of N and are shown in Table-1. The bounds obtained by Arthurs and Arthurs [6] are -1.049 and -1.077.

Approximate values of $u'(0) = a$

N	a
4	-0.7293455514
8	-0.8041111126
12	-0.8695980508
16	-0.9244272505
20	-0.9709162561
24	-1.000515226
28	-1.021515903

Table-1

For various N, we obtainthe analytic solution and numerical values obtained in TEA and ADMare shown in Table-2.

Comparison between TEA and ADM

x	TEA		ADM	
	N=24	N=28	S_3	S_4
0.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.1	0.9093353697	0.9072222507	0.90785296	0.90494962
0.2	0.8353624148	0.8310639253	0.83234060	0.82644116
0.3	0.7756278161	0.7690120212	0.77092409	0.76189632

0.4	0.7283558697	0.7192383989	0.72166066	0.70942016
0.5	0.6922618868	0.6804081289	0.68297597	0.66758526
0.6	0.6664322386	0.6515557422	0.65352893	0.63527429
0.7	0.6502471011	0.6320052284	0.63214522	0.61155985
0.8	0.6433193295	0.6213191156	0.61780086	0.59561289
0.9	0.6451589572	0.6191496182	0.60964089	0.58663566
1.0	0.6507707604	0.6222708133	0.60702147	0.58382711

Table-2

From the table, it can easily be seen that ADM converges rapidly than TEA. For large values of N, the approximate solution obtained in TEA is very close to the solution obtained in ADM.

Conclusion

In this paper, Taylor Expansion Approach has been applied to solve nonlinear integro-differential equation arising in cell membrane theory. This method is applicable to all problems in the general form. This method transformed the nonlinear integro-differential equations to a matrix equation which corresponds to a system of nonlinear equations with unknown coefficients. Finally, by using this system, we find the approximate solution of the nonlinear integro-differential equation and comparison is made with ADM. All calculations were done using Maple.

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